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A		2610126

A Modular Framework for Smartphone Battery SOC and Endurance Prediction: Integrating Differential Equations with User Behavior Clustering

Summary

To accurately estimate **smartphone State of Charge (SOC)** and **predict endurance**, this study establishes a **modular mathematical framework**. We first ensured dataset integrity via outlier removal and standardization. Pearson correlation analysis revealed high variable independence, validating a decoupled modeling approach.

Model I establishes a **Continuous-Time SOC Model** to characterize battery depletion using Kirchhoff's Current Law and differential equations. An enhanced Ampere-Hour (A-H+) formulation is developed by incorporating a Battery Aging Model, which captures a thermodynamic subsystem describing temperature-dependent internal resistance variations. Simulation results demonstrate that the model maintains high fidelity in tracking SOC evolution under diverse usage scenarios and operational conditions.

Building upon the SOC framework, **Model II** introduces a **Time-to-Empty (TTE) Prediction Model** to forecast remaining usable time. User behavior patterns are categorized via K-Means clustering, and a matrix mapping is constructed to relate hardware-level power consumption to application usage. The model achieves accurate TTE prediction and identifies the dominant contributors to rapid battery depletion.

To address long-term storage scenarios, **Model III** presents a **Static Idle Power Run Out Model** that integrates battery self-discharge and calendar aging mechanisms. This model quantifies charge loss and irreversible capacity degradation during dormancy, thereby providing a theoretical basis for scientifically grounded battery storage guidelines and mitigation of hardware obsolescence risks.

Finally, **sensitivity analyses** using the Morris Method and Tornado Diagrams reveal that initial battery capacity exerts the strongest influence on endurance outcomes, while Monte Carlo simulations verify the robustness and stability of model predictions under parameter uncertainty. Based on these findings, practical recommendations are proposed, including optimizing screen luminance settings and integrating the modeling framework into Battery Management Systems (BMS) to enhance endurance prediction accuracy and extend device longevity.

Keywords: Smartphone lithium-ion battery; Continuous-time modeling; State of Charge (SOC); Time-to-Empty (TTE); User behavior clustering

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1 Introduction

The core objective of this mathematical model is to establish a framework that continuously characterizes the variation of the battery's State of Charge (SOC) over time, and subsequently predicts the remaining available usage time under various usage scenarios.[1]

In many electronic devices, battery SOC data is estimated based on voltage or obtained using a Coulomb counter. Voltage-based data is often inaccurate due to changes in internal battery resistance over time and fluctuations in current; while Coulomb counting is precise, it incurs higher costs. Therefore, this method allows for SOC detection and endurance prediction without increasing hardware costs, volume, or weight. This facilitates user decision-making regarding whether to charge the device or carry a portable power bank based on the predicted endurance.

The crux of this problem lies not merely in fitting the battery depletion curve, but in constructing a continuous-time model with physical and engineering significance that can explain why battery consumption rates differ significantly across different contexts. This model can derive the mobile phone's SOC and predict remaining endurance based on the device's hardware parameters and historical user interaction data. Furthermore, for long-term usage, it can rationally allocate mobile performance and sleep/standby intervals based on user habits to reduce battery consumption, while providing users with recommendations for battery protection over the device's life-cycle.

1.1 Problem Background

Battery capacity determines the remaining operational time of mobile electronic devices. Therefore, modeling power consumption behavior and analyzing discharge patterns can effectively reduce battery consumption and extend the usage time of a single charge. This, in turn, optimizes charging frequency, mitigates the reduction of cycle life, and ultimately extends the overall service life of the mobile phone.

Simultaneously, in an era where electronic devices are becoming increasingly thin and lightweight, the battery has become a critical constraint on weight and volume. Through this model, devices with smaller batteries can achieve endurance performance comparable to those with larger batteries. This presents significant benefits regarding cost constraints, portability, and user experience.

We focus on the relationship between remaining battery duration and user behavior. By analyzing the mobile phone as a holistic system and incorporating Human-Computer Interaction (HCI) into our considerations, we believe that approaching the problem from the user's perspective can effectively enhance the robustness and universality of the model.

1.2 Restatement of the Problem

Considering the background information and restricted conditions identified in the problem

statement, we need to solve the following problems:

-Problem 1: How to distinguish between different power consumption behaviors and construct a mathematical model for them?
-Problem 2: How to correlate distinct power consumption behaviors (e.g., the relationship between environmental temperature and battery performance, or ambient light and screen brightness)?
-Problem 3: How to construct a feasible method to predict future power consumption behaviors based on historical data?
-Problem 4: How to conduct data analysis and model tuning?
-Problem 5: How to provide reasonable and effective usage recommendations based on the model data?

1.3 Literature Review

Lithium-ion batteries (LIBs) have become the preferred choice for many power-consuming devices due to their high energy density, long cycle life, compact volume, light weight, and good safety performance. [2] As a key technology in battery systems, the Battery Management System (BMS) plays an indispensable role; however, improvements are still needed in areas such as cell balancing and data reliability when accurately estimating SOC.[3]

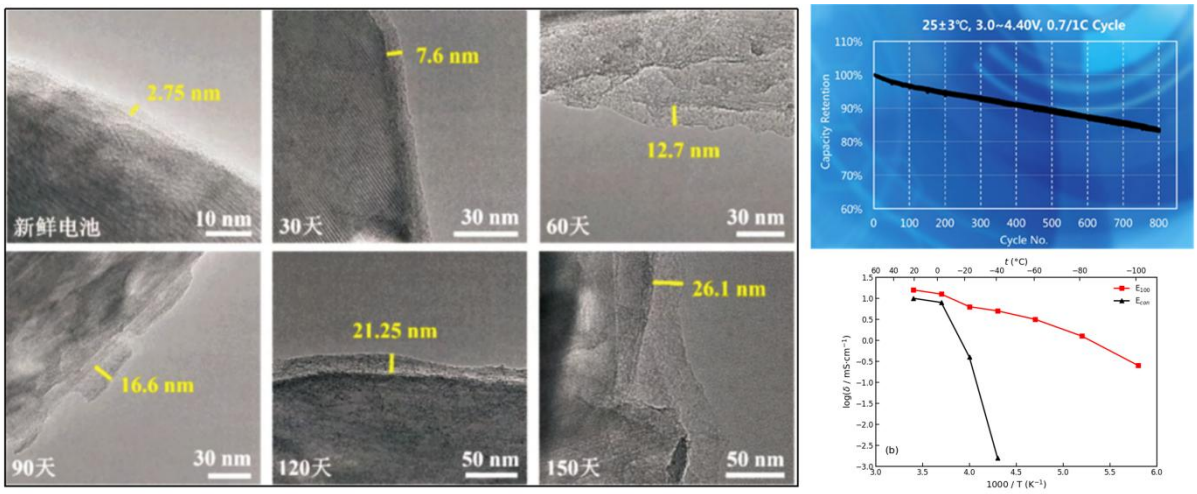


Figure 1

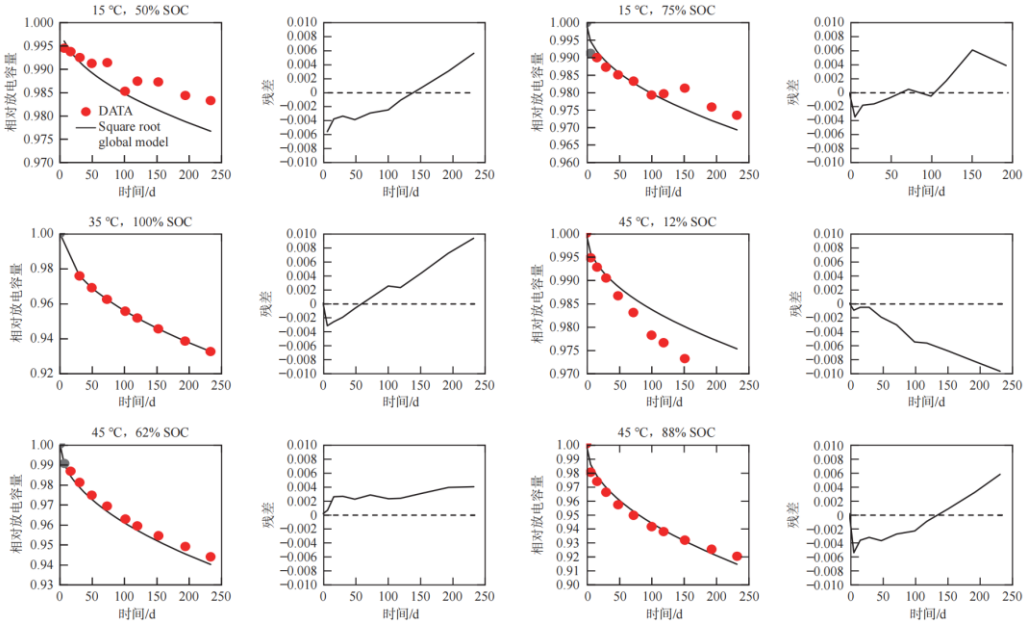
Left 100% SOC stage storage from 0d to 150d.[4]

Right top General mobile phone battery capacity with cycle number, showing an approximately linear downward trend after the first 100 cycles. [5]

Right bottom Ionic conductivity of the Low-temperature electrolyte E−100 from −100 ℃to 25 ℃, and conventional electrolyte E−con from −40℃ to 25℃. [6]

The experimental results in Figure 2 demonstrate a sharp decline in the performance of conventional electrolytes at low temperatures. Consequently, we have introduced the influence of temperature on battery performance into our model: specifically, **internal resistance**.

In terms of calendar aging, the dominant factor is the degradation caused by the slow thickening of the anode film. Transmission Electron Microscopy (TEM) tests were conducted every 30 days on identical battery samples stored at 100% SOC, with results shown in Figure 1 The Solid Electrolyte Interphase (SEI) film thickness of a fresh battery was 2.75 nm. As the aging time increased, the SEI film thickness grew gradually, showing a roughly linear growth trend. After 150 days of aging, the SEI film thickness reached 26.1 nm. As the SEI film thickens, the lithium-ion transport channels within the film narrow, limiting the rate of intercalation and deintercalation of lithium ions from the electrolyte to the electrode. Restricted lithium-ion transport slows down the battery's charge and discharge speeds and leads to capacity fading. Therefore, slowing the growth rate of the SEI film and maintaining its stability are key to extending the life and improving the performance of lithium-ion batteries.

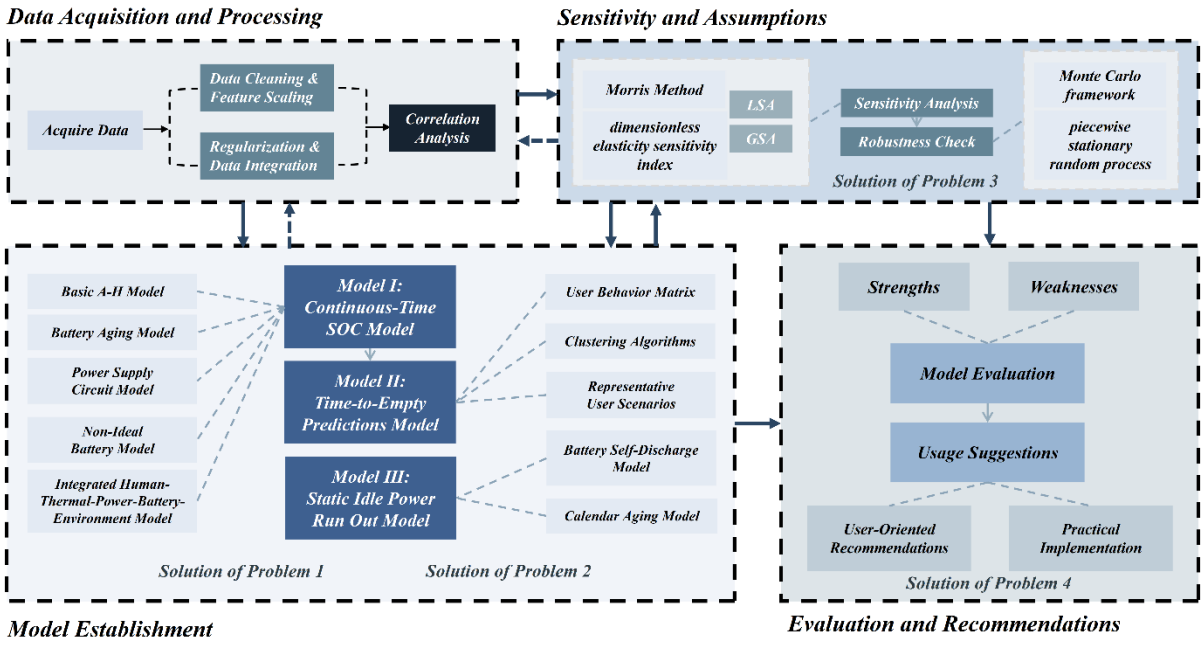


Accurately predicting battery aging behavior during long-term storage is equally critical. Since battery capacity decreases with usage time, SOC estimation cannot rely indefinitely on the initial battery capacity; therefore, we must introduce a **battery aging model** [7].

Methods of SOC Estimation: SOC estimation methods have evolved from the initial Open Circuit Voltage (OCV) method and the Ampere-hour integral method (Coulomb counting) to the more recent Kalman filter estimation models and neural network algorithms [8].

Note regarding the constraints of this problem: Kalman filtering and neural network algorithms are not permitted (as they involve "black box" neural networks or are unable to explainably predict non-linear user operations).

1.4 Our Work



2 Assumptions and Justifications

[Assumption 1] We assumed that the discharge process does not affect the battery's total capacity during a single discharge cycle; only upon full recharge does the total capacity change.
[Justification 1] The capacity loss of the battery constitutes a small quantity relative to the total capacity over a charge-discharge cycle. In other words, the battery's total capacity is treated as a discrete variable related to cycle count, remaining constant within a discharge cycle.

[Assumption 2] the supply voltage to the mobile phone's motherboard via the DC-DC converter is assumed to remain constant
[Justification 2] Because DC-DC converters have feedback circuits, under normal operating conditions and do not change with decreasing battery voltage.

[Assumption 3] Only the battery internal resistance and battery self-discharge currents are considered under varying temperatures, assuming constant power supply circuit components.
[Justification 3] Because temperature-induced changes in power supply circuit component characteristics are considerably smaller than those in battery internal resistance.

[Assumption 4] We assume that for the vast majority of users, behavior is similar when using the same software.
[Justification 4] The differing functionalities of applications lead to distinct usage patterns, such as we won't use online chatting apps as game apps.

[Assumption 5] we assume that the contribution of background apps, foreground apps, cameras, etc., to instantaneous current forms a stationary sequence over a short duration.

[Justification 5] As the current peak during application initialization is transient, during periods of stable user behavior, the average effective value can be seen as fixed.

[Assumption 6] During intervals where user behavior remains relatively stable and shows no significant variation (active times, usually less than 2 days), we assume that charge loss due to self-discharge can be considered negligible. (compared to Assumption 11)
[Justification 6] The duration is too short to let battery self-discharge show up.

[Assumption 7] When SOC falls below a preset threshold (probably 5%), users intentionally (or system-forced) cease using power-consuming components (e.g., foreground/background apps, camera, touchscreen interactions). At this point, self-discharge can no longer be ignored, especially when the phone is not used for a long time (more than a month), but the screen remains on at minimum brightness while active (Typically, this takes place when the smartphone is forcibly awakened.).

[Assumption 8] The mobile phone does not exert any measurable influence on the ambient temperature; the thermal condition of the user's palm remains approximately constant; and the device exhibits a relatively uniform heat transfer coefficient.

3 Notations and Definitions

Table 1: Notations used in this paper

Symbol	Description	Unit
Q	Current battery capacity (charge)	mAh
Q_0	Battery capacity at the last full charge	mAh
Q_{fc}	Initial full-charge capacity	mAh
Q_h	Lifetime charge throughput	mAh
$I_{battery}$	Total battery output current	mA
SOC	State of Charge (remaining capacity percentage)	/
E	Electromotive force (EMF) of the battery	mV
r	Internal resistance of the battery	mΩ
I_o	Total current of the power supply circuit	mA
I_f	Current consumed by foreground applications	mA
I_{bi}	Current consumed by the i-th background application	mA
I_m	Touch-based interaction current	mA
I_{sc}	Screen backlight current	mA
I_c	Camera operational current	mA
I_{st}	Quiescent (static loss) current	mA
K_0	Absolute processor energy efficiency	Mark/W
K	Relative processor energy efficiency	-
U_o	Output voltage of the power supply circuit	V
cd	Screen luminance (brightness)	nit
W	Screen width	inch
H	Screen height	inch

Symbol	Description	Unit
ppi	Pixel density (pixels per inch)	pixel/inch
k	Time-dependent aging coefficient	-
l	Unit charge-induced degradation factor	-
t_0	Time elapsed since the last full charge	s
t	Battery age (total time since made)	s
T	Battery temperature	℃
T_{air}	Ambient air temperature	℃
T_{palm}	User's palm temperature	℃
P_p	Thermal power generation	W
P_d	Thermal power dissipation	W
I_{sd}	Self-discharge currents	mA

Table 2: Definitions

Symbol	Definition
SOC	State Of Charge
SoC	System on Chip

To avoid ambiguity, SOC is used to denote the state of charge, whereas SoC refers to a system on chip.

4 Data Description

4.1 Data Pre-processing

To ensure the integrity and robustness of our model, the raw dataset underwent a rigorous pre-processing pipeline. The steps are outlined as follows:

• **Outlier Removal:** We identified and excluded data records containing anomalies or non-physical values that could skew the model's parameters.

• **Boundary Data Treatment:** "Tail-end" data points were processed to mitigate noise and instability often found at the beginning or end of discharge cycles.

• **Correlation Analysis:** Preliminary checks were conducted to identify redundancy among features.

• **Normalization and Standardization:** To eliminate the influence of different physical units and scales, the data were scaled using **Z-score standardization** and **Min-Max normalization**, ensuring all features contribute proportionately to the model.

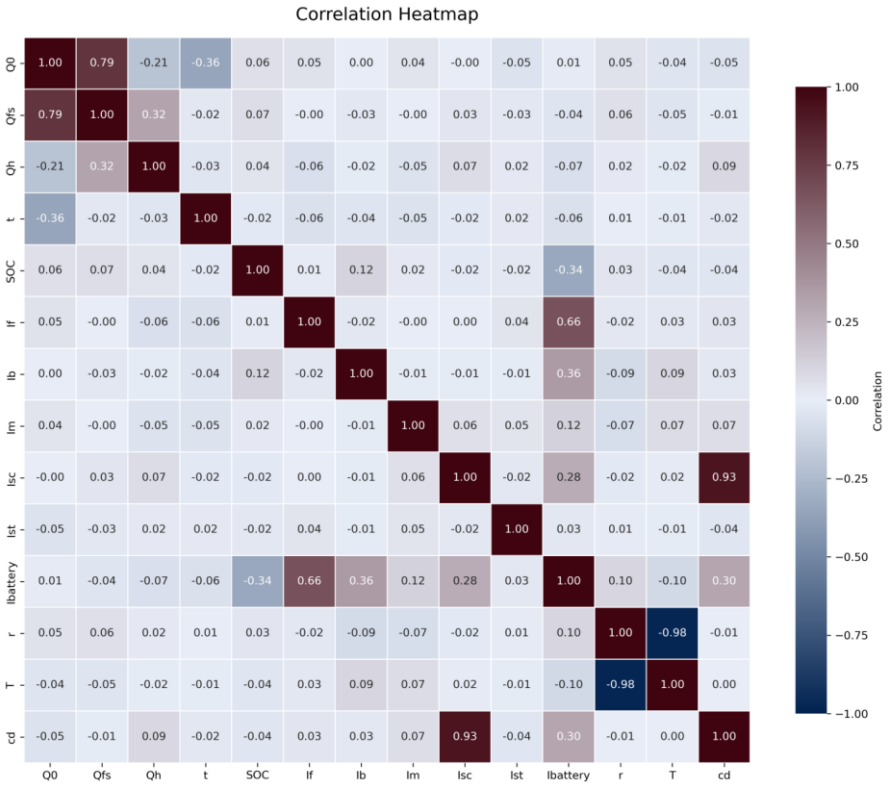
• **Visual Inspection: Boxplots** were generated to visualize the distribution of key variables and verify the effectiveness of our outlier detection.

4.2 Data Correlation Analysis

We performed an Exploratory Data Analysis (EDA) by generating a **Heat-map** to evaluate the Pearson correlation coefficients between all measured variables and SOC. This analysis serves two primary purposes:

• **Feature Selection:** Identifying the variables that most significantly characterize the dynamics of SOC depletion.

• **Structural Validation:** The heat-map reveals that the correlations between different input parameters are remarkably low, indicating that the variables are nearly independent.



The near-independence of the variables provides a rigorous statistical foundation for our **Modular Modeling Approach**, allowing us to model the power consumption of individual components separately before aggregation.

5 Model I: Continuous-Time SOC Model

5.1 The Establishment of Continuous-Time SOC Model

To establish a power consumption model and fulfill the requirement of characterizing the energy behavior of various smartphone components, we develop a differential equation-based framework. By applying Kirchhoff's **Current Law (KCL)**, the total current is derived by summing “sub-currents”.

Furthermore, we adopt the classical **Ampere-Hour (A-H) method** to calculate power depletion. This approach effectively avoids the "black-box" limitations inherent in neural network algorithms and bypasses the complexities that Kalman filtering faces when processing nonlinear user operations. While traditional A-H methods integrate current to obtain total energy consumption, we propose an enhanced **A-H+ method** based on differential equations. This modified method is specifically designed to handle the multi-component, human-machine interaction states of modern smartphones, preserving the robustness of the traditional A-H approach while improving its precision in dynamic environments.

5.1.1 Variable Screening and Simplification

A. Through empirical testing and rigorous validation against our datasets, we conducted a significance analysis to refine our model. The results indicate that the following metrics exert **no statistically significant influence** on power consumption:

1.Audio Volume Level

2.The charge consumption of the rest apps (the apps not included in "Streaming apps, music players, search engine apps, social media apps, file download and management apps, Gaming apps ") in the background is negligible.

B. Beyond the excluded variables, our empirical testing and dataset validation reveal that **several key indicators exhibit distinct behavioral patterns.** These patterns provide the mechanistic basis for our differential equations and help characterize the complex dynamics of battery depletion:

1.Streaming apps, music players, search engine apps, social media apps, file download and management apps have similar charge consumption rates in both foreground and background states. Gaming apps consume significantly more power in the foreground than in the background.

2. The primary behaviors causing phone power consumption are inherent hardware power consumption, foreground apps, background apps (including system resources required by apps), finger-screen interactions, and screen illumination.

3. Different apps are assumed to be working independently when system resources are left.

4. Based on our testing data, data services such as calls and Wi-Fi have a significantly smaller impact on charge consumption rate compared to other factors (e.g., active apps, camera usage, screen activity). So, we assume that calls and Wi-Fi contribute a static low current to the system.

5. Based on our testing data, finger-screen interactions with the smartphone, including taps for typing and swipes for scrolling, result in an almost fixed increment of current.

5.1.2 Basic A-H Model: The Computational Foundation

As the cornerstone of the entire system, the basic A-H model integrates instantaneous current to determine the cumulative charge consumed. The remaining battery capacity is expressed as:

$$Q = Q_0 - \int_0^{t_0} I_{battery}\, dt \tag{1}$$

5.1.3 Battery Aging Model

Over time and through repeated charge-discharge cycles, lithium-ion batteries exhibit aging phenomena such as the thickening of the SEI film and lithium-ion loss, which resulting in increased internal resistance and capacity degradation respectively. To quantitatively describe this non-linear degradation behavior, we construct a **Calendar-Cycle Aging Model** that relates the battery's effective full capacity to cumulative charge loss. The degradation process is decomposed into two complementary mechanisms:

- **Calendar-based degradation**: Even completely unused, a smartphone's maximum battery capacity will degrade to k times its original value (k<1) after each duration, following an exponential pattern.
- **Cycle-based degradation**: In addition to calendar aging, historical charge–discharge cycles

further accelerate capacity loss. Repeated cycling induces irreversible structural and chemical changes within the battery, resulting in an exponential degradation of the maximum available capacity.

Accordingly, the relationship between the effective full-charge capacity and cumulative charge throughput is expressed as

$$Q_0 = Q_{fs} \cdot e^{-kt - l \cdot Q_h} \tag{2}$$

where the lifetime charge throughput is defined by

$$Q_h = \int_0^t I_{battery}(\tau)\, d\tau \tag{3}$$

Based on real data from the experimental database and dataset fitting, the estimated values for the constants are as follows: [9]

$$l = 5.7751 \times 10^{-5}$$

5.1.4 Power Supply Circuit Model

In smartphones, the system motherboard is typically powered by a Step-down Voltage Regulator (Buck Converter) rather than directly by the lithium-ion battery. This configuration ensures voltage stability for downstream circuits and enhances the reliability of electronic components. Leveraging this characteristic together with KCL, we construct a power supply circuit model to characterize the relationship between individual branch currents and the total load current. Specifically, the load current drawn by the external circuit is expressed as

$$I_o = \left(I_f + \sum I_{bi}\right) \cdot K + I_m + I_{sc} + I_{st} \tag{4}$$

where the total output current is decomposed into contributions from foreground applications, background applications, touch-based interactions, screen backlight, and static losses.

To account for the non-linear efficiency characteristics of the processor, the absolute processor efficiency coefficient is modeled as

$$K_0 = \frac{\alpha_1 \cdot \left(1 - e^{-\beta_1 \cdot I_o}\right)^{\gamma_1}}{I_o} \tag{5}$$

which captures the behavior of processor efficiency as the load current increases.

The screen backlight current is further modeled as a function of screen-relevant parameters:

$$I_{sc} = \left(\alpha \cdot W \cdot H \cdot \left(\frac{ppi}{\beta}\right)^{\gamma}\right) \cdot \frac{cd}{1 + \delta \cdot (cd)^2} \tag{6}$$

where screen dimensions, pixel density, and brightness jointly determine the power consumption of the display subsystem.

Based on real data, the estimated values of the model constants are given as follows:

$$\alpha_1 = 3415.9188,\ \beta_1 = 0.5554119,\ \gamma_1 = 1.0764587$$

$$\alpha_2 = \frac{0.0691mA}{cd},\ \beta_2 = 326\left(\frac{pixel}{m}\right)^{0.5},\ \gamma_2 = 0.5,\ \delta_2 = 1.8564 \times 10^{-6} nit^{-2}$$

5.1.5 Non-Ideal Battery Model

A physical battery is not an ideal power source; it possesses internal resistance, and its terminal voltage drops during discharge. In addition, the internal resistance fluctuates with temperature and SOC. We model the interdependencies between Temperature-Internal Resistance, SOC–

Voltage, Current-Ohmic Loss, and SOC-Capacity. This **non-ideal battery model** enables an accurate description of battery performance under practical operating conditions.

The battery current and the associated state transitions are governed by the following relations:

$$\begin{cases} \left(E - I_{battery} \cdot r\right) \cdot I_{battery} = U_o \cdot I_o & (7) \\ r = r_{25} + AT^2 + BT + C & (8) \\ SOC = \left|\dfrac{Q}{Q_0}\right| & (9) \\ E = E_{min} + (E_{max} - E_{min}) \cdot SOC & (10) \end{cases}$$

Based on real data, the estimated values of the model constants are given as follows:

$$r_{25} = 323.0m\Omega,\ A = 0.147616m\Omega/℃,\ B = -13.299756m\Omega/℃,\ C = 235.420058m\Omega$$

$$E_{max} = 4.3,\ E_{min} = 3.3$$

5.1.6 Integrated Human-Thermal-Power-Battery-Environment Model[10]

Based on thermodynamic principles, we model the smartphone as **an integrated system** to capture the characteristic evolution of battery behavior under the combined influence of temperature, environment, and user interaction. This framework provides an improved description of battery depletion across varying the smartphone and ambient temperatures, usage scenarios, and operational intensities.

The governing equations for the integrated model are given as follows :

$$\begin{cases} P_p = U_o \cdot I_o + I_{battery} \cdot r & (11) \\ \mathrm{P_d} = -\mathrm{h_{air}} \cdot \mathrm{W} \cdot \mathrm{H} \cdot (\mathrm{T} - \mathrm{T_{air}}) - \mathrm{h_{palm}} \cdot 0.5 \cdot S \cdot \left(\mathrm{T} - \mathrm{T_{palm}}\right) & (12) \\ C \cdot \mathrm{M} \cdot \dfrac{\mathrm{dT}}{\mathrm{dt}} = P_d + P_p & (13) \end{cases}$$

Here, P_p represents the total thermal power generated by electrical operation, including both external circuit power and ohmic losses within the battery. The term P_d models heat dissipation to the surrounding air and the user's palm through convective heat transfer, while the energy balance equation governs the temporal evolution of the device temperature.

5.2 The Solution of Continuous-Time SOC Model

Leveraging our established mathematical model and a Python-driven visualization framework, we generated a continuous profile illustrating the variation of SOC over time under predefined environmental constraints.

Initial States

Parameter	Value	unit
Qf_s	5785	mAh
t	31536000 （~1 year）	s
Q_h	898885（~160cycles）	mAh
W	2.7	inch
H	5.9	inch
ppi	400	pixel/inch
$Top\ brightness$	500	nit
T_{air}	20	℃

To evaluate the impact of user behavior on battery depletion, three representative usage profiles are defined:

- **Saver User**: Characterized by minimized screen brightness, infrequent user-screen interaction, and the avoidance of high-power tasks (e.g., gaming, streaming media, or camera usage).
- **Normal User:** Represents the median software usage distribution derived from the data set, with standard brightness settings and moderate multitasking.
- **Heavy User**: Defined by high screen brightness, frequent touch interactions, and a workload dominated by energy-intensive applications such as 3D gaming, high-definition streaming, and large-file downloads.

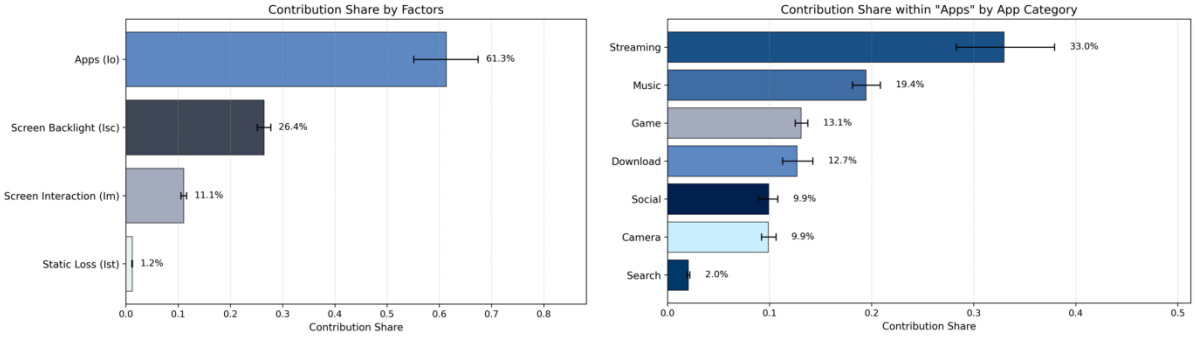


Figure 3:

(Left) Detailed Decomposition of Power Consumption by Hardware Components in Model I

(Right) Detailed Breakdown of Power Consumption across Application Categories in Model I

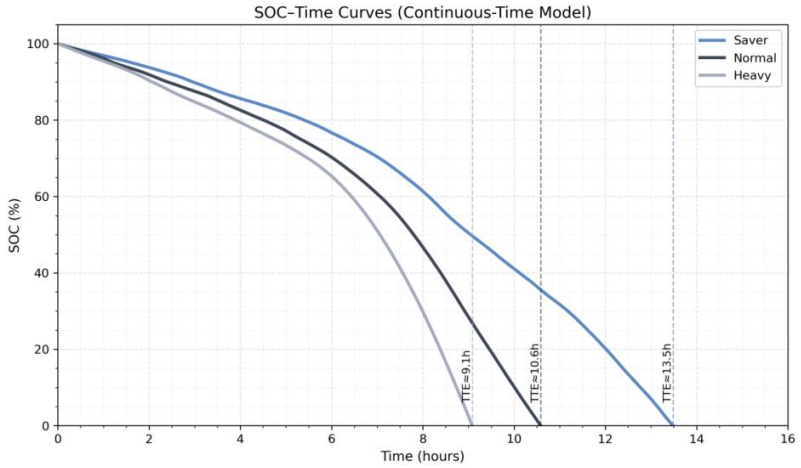


Figure 3: Continuous-Time SOC Line Chart

The simulated results exhibit high fidelity to real data and existing benchmark datasets, confirming that the proposed model accurately captures charge consumption under practical operating conditions.

6 Model II: Time-to-Empty (TTE) Prediction Model

6.1 The Establishment of Time-to-Empty Predictions Model

Having established the Continuous-Time SOC Model, we must now address the critical question: "When will the battery be fully depleted?"

By integrating historical user behavior patterns with the mathematical characteristics of the battery established in the previous section, we employ a **clustering algorithm** to categorize user behavior types. This approach allows us to define specific feature states, enabling us to predict the remaining usage time more accurately by fitting the model to specific user scenarios.

6.1.1 Matrix-Based User Profiling Method

We decompose the smartphone's primary dynamic power consumption into three fundamental hardware components: the Camera, the System on Chip (SoC), and the Screen. Since different applications utilize the resources of these three components in varying quantities, the relationship between the underlying hardware and the application set can be modeled as a linear combination.

Furthermore, user behavior can be regarded as a linear combination of the application set. By transitivity, this formulation yields a direct mapping between the underlying hardware components and user behavior. As a result, each user can be uniquely located within a three-dimensional coordinate system defined by the Camera, SoC, and Screen dimensions.

Mathematically, the application set is expressed as

$$(\text{apps}) = \left(a_1 \begin{pmatrix} cam_1 \\ SoC_1 \\ screen_1 \end{pmatrix}, a_2 \begin{pmatrix} cam_2 \\ SoC_2 \\ screen_2 \end{pmatrix}, a_3 \begin{pmatrix} cam_3 \\ SoC_3 \\ screen_3 \end{pmatrix} \ldots \right) = \begin{pmatrix} cam_1 & \cdots & cam_n \\ SoC_1 & \ddots & SoC_n \\ screen_1 & \cdots & screen_n \end{pmatrix}$$

and the user behavior vector is given by

$$(user) = (b_1 \quad b_2 \quad b_3 \quad \ldots b_n) \begin{pmatrix} cam_1 & \cdots & cam_n \\ SoC_1 & \ddots & SoC_n \\ screen_1 & \cdots & screen_n \end{pmatrix}^T = \begin{pmatrix} cam_{sum} \\ SoC_{sum} \\ screen_{sum} \end{pmatrix}^T$$

6.1.2 User Behavior Analysis Based on Clustering Algorithms

We employ the **K-Means clustering algorithm** to perform unsupervised classification on the data, yielding four mutually exclusive clusters. The objective of this algorithm is to maximize intra-cluster similarity while maximizing inter-cluster variance. This is mathematically achieved by minimizing the sum of squared Euclidean distances between sample points and their respective cluster centers.

6.1.3 How does it work?

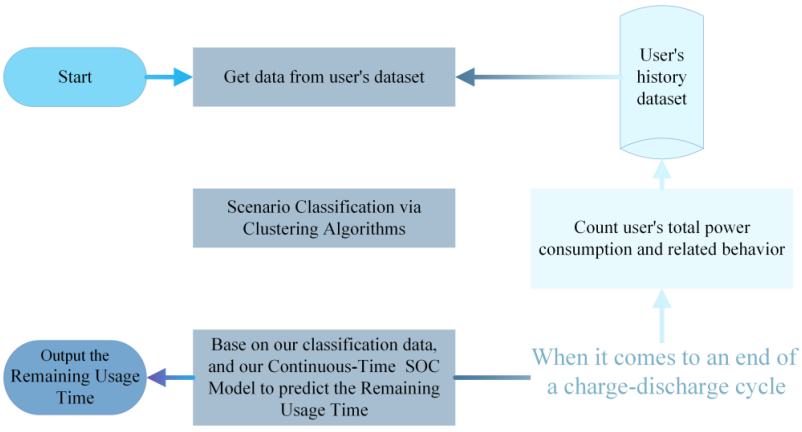


Chart: A flow chart of how the model works for different users

6.2 The Solution of TTE Model

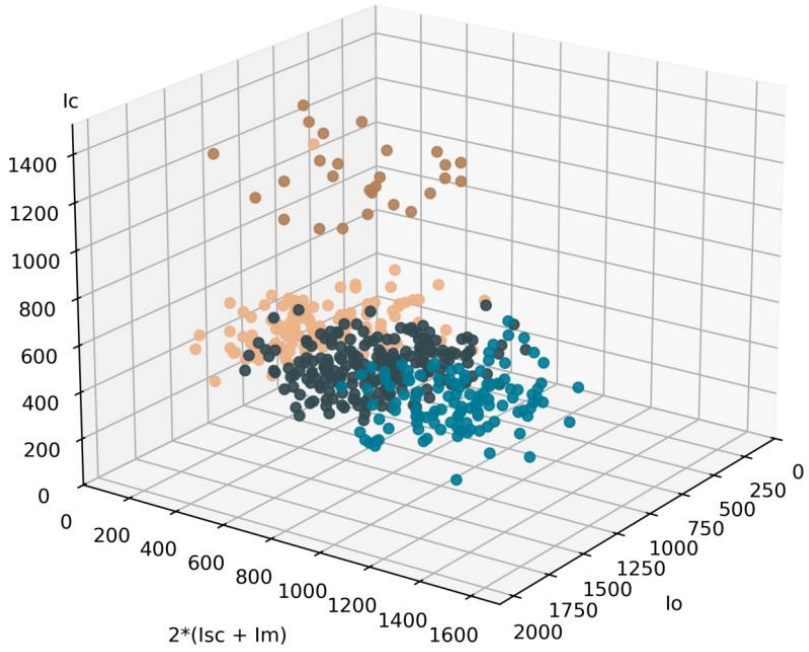


Chart : Visualization of the three-dimensional coordinate distribution processed by the clustering algorithm

Based on the modeling information extracted through the clustering analysis, we characterize the distinct usage patterns associated with different user types. By mapping the model parameters to these specific user profiles, we present the following simulation results and predictive outcomes.

Parameter Assignments for Representative User Scenarios:

	Q_{fs}	Q_0	T	W*H	ppi	cd	I_m	I_{st}	I_o	K	Cycle	t(s)
A	5800	4687.14	25	3.0*6.5	566	375	58.22	12	904.79	2.22	203	31363200
B	7200	5656.18	15	2.8*6.1	441	436	140.33	10	612.31	2.50	252	30240000
C	5000	4106.51	30	2.7*5.9	441	229	52.63	9	885.24	2.27	175	27820800
D	8600	6567.49	5	2.5*5.5	411	260	36.42	11	367.92	2.5	301	34992000

Predicted TTE Results Across Varying Initial SOC Levels

	100%	80%	60%	40%
Case A	2.57	2.06	1.54	1.03
Case B	6.92	5.54	4.15	2.77
Case C	4.37	3.49	2.62	1.75
Case D	10.37	8.29	6.22	4.15

6.3 Analysis of the Causes of Rapid Battery Drain

To provide a robust explanation for the variance in power consumption across different scenarios, our model decomposes the total discharge rate into specific hardware and behavioral drivers. The following analysis identifies the primary factors leading to rapid drain in each of the four characteristic cases:

Case A: High-Definition Multimedia & Heavy Recording (Vlogger/Traveler)

Case B: Media Streaming & Social Engagement (Streamer/Lover)

Case C: High-Performance Computing (Gamer/Heavy Worker)

Case D: Low-Intensity Usage (Light User/Student)

7 Model III : Static Idle Power Run Out Model

7.1 The Establishment of Static Idle Power Run Out Model

Mobile devices are not always in active use; however, long-term storage often leads to irreversible battery degradation, commonly referred to as "dormancy failure." Given that modern smartphones typically employ non-removable batteries, a complete battery failure can result in permanent data loss and hardware obsolescence. To address this issue, we established a **Static Idle Power Run Out Model** to simulate battery health during storage and to provide users with scientifically grounded storage guidelines.

This model integrates the previously defined Battery Self-Discharge Model and Calendar Aging Model to characterize long-term static power loss. The core modeling logic accounts for the fact that, even when the device is powered off, the battery continues to lose charge due to internal chemical reactions as well as the gradual thickening of the SEI film.[11] The governing equation of the Static Idle Power Run Out Model is expressed as

$$\frac{dQ}{dt} = -I_1 e^{-\frac{E_a}{R \cdot T}} - e^{\beta \cdot SOC_0} + I_2 \tag{14}$$

where the battery self-discharge current I_{sd} follows the Arrhenius law:

$$I_{sd}(T) = I_1 \cdot e^{-\frac{E_a}{RT}} \tag{15}$$

Variable Definitions:

I_{sd}: Battery self-discharge current (mA).

E_a: The activation energy of the cathode reaction (J/mol).

R,I_1: Constants derived from empirical data fitting.

$\Delta I_{aging}(SOC)$: Battery capacity degradation induced by specific SOC levels.

I_2: Parasitic leakage current flowing to the protection circuit board and other dormant components (mA).

7.2 The Solution of Static Idle Power Run Out Model

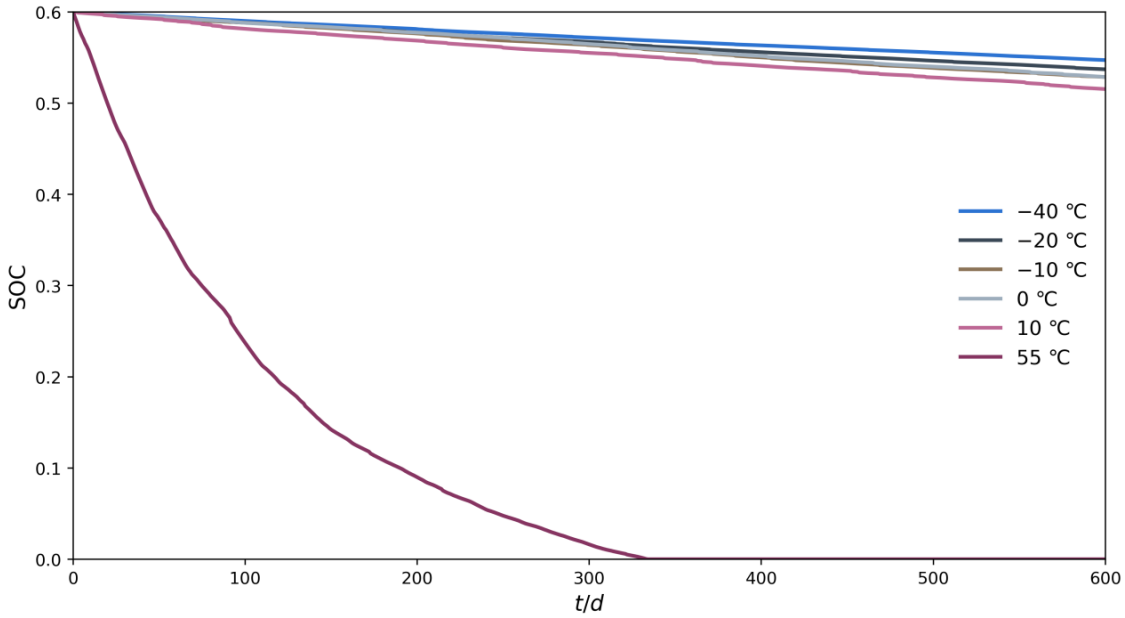


Figure 4: Simulation of Long-term SOC Decay under Off-state Storage at Different Temperatures

The simulation results align with observed empirical phenomena: storage temperature is a critical determinant of battery health. High-temperature environments significantly catalyze the internal chemical reactions (I_{sd}), leading to a rapid decline in capacity. This accelerated degradation not only depletes the charge but also risks structural damage to the lithium-ion cells, potentially rendering the device unrecoverable.

8 Sensitivity and Robustness Analysis

8.1 Sensitivity Analysis

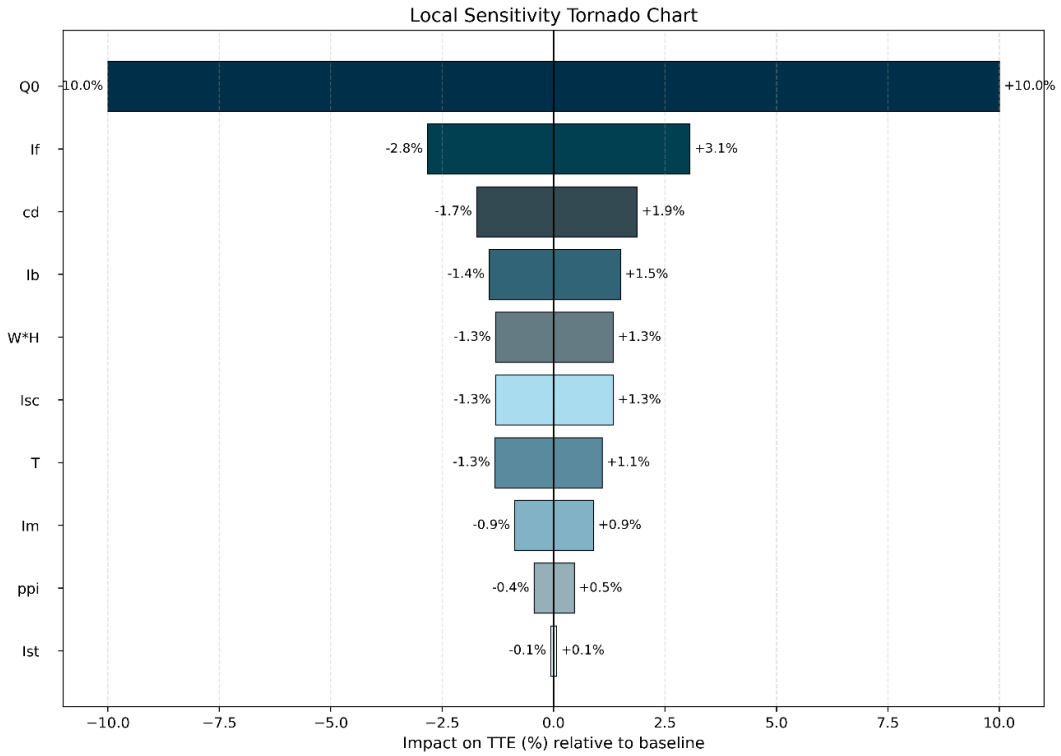
To validate the reliability of our model, both local and global sensitivity analyzes are performed.

8.1.1 Local Sensitivity Analysis

We select **Scenario B** as the baseline for the local sensitivity analysis. For each input parameter p, we apply a relative perturbation of $\pm\delta$ (where $\delta = 10\%$) while keeping all other variables constant to observe the resulting variation $[TTE(p(1 \pm \delta))]$ in the output. We utilize the **dimensionless elasticity sensitivity index**, defined as follows:

$$S_p = \frac{\Delta TTE/TTE}{\Delta p/p}$$

A **Tornado Diagram** is employed to visualize the magnitude of the impact of major parameters on the Time-to-End (TTE).



In this diagram, a positive direction indicates that an increase in the parameter leads to an increase in TTE, while a negative direction implies the inverse.

This analysis effectively identifies the dominant factors affecting battery endurance and provides a solid foundation for the subsequent robustness analysis and model simplification.

8.1.2 Global Sensitivity Analysis (Morris Method)

To evaluate the model's sensitivity across the entire parameter space, we adopted the **Morris Method**. Multiple random trajectories were sampled within the permissible range of each parameter. Along each trajectory, discrete perturbations were sequentially applied to single parameters to derive the "Elementary Effect" (EE) of that parameter on the output:

$$EE_i = \frac{TTE(x + \Delta e_i) - TTE(x)}{\Delta}$$

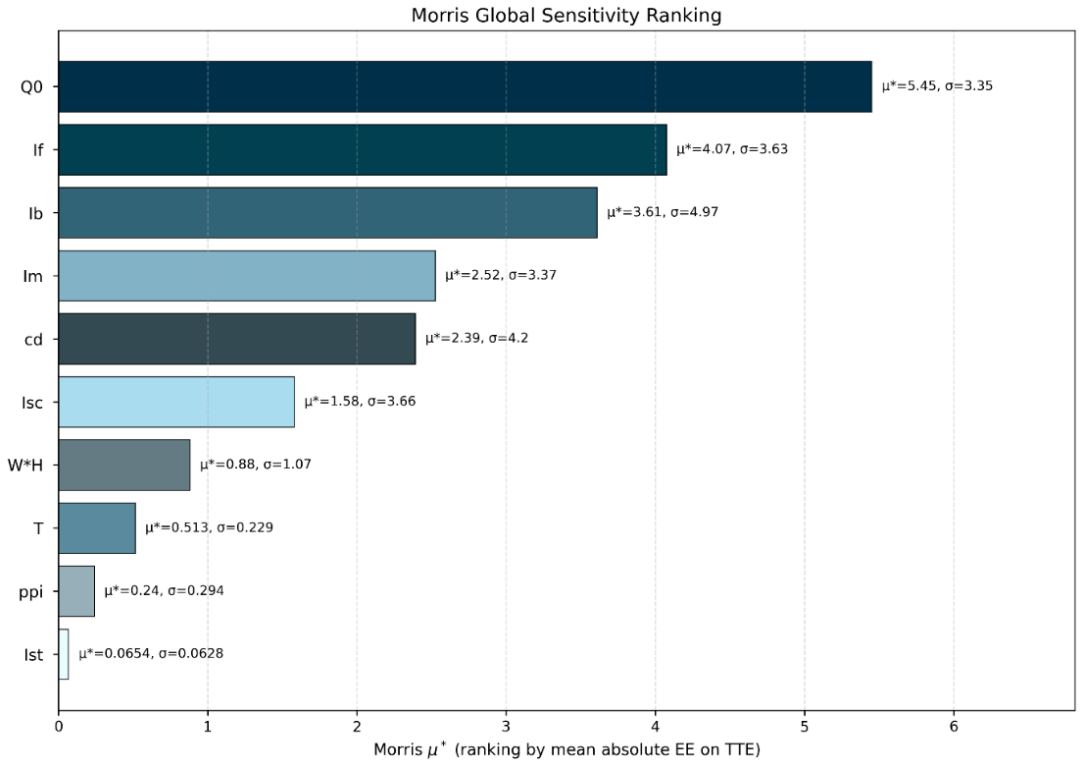
Where x represents the normalized parameter vector, e_i denotes the unit vector in the direction of the i-th parameter, and Δ is the fixed discrete step size.

For each parameter, a set of EEs was obtained across multiple random trajectories. We calculated the mean (μ^*) and standard deviation (σ) to measure the following:

- **Mean (μ^*):** Measures the overall influence intensity of the parameter on the output. Parameters with the highest mean rankings are considered the dominant factors affecting TTE.
- **Standard Deviation (σ):** Measures the instability of the parameter effect. A large σ suggests that the parameter's influence varies across different regions, typically implying **non-linearity** or **interaction effects** with other parameters.

Consequently, parameters with both small means and standard deviations are deemed to have

limited impact on TTE within the given range, justifying their exclusion for model simplification or reduced measurement precision requirements.



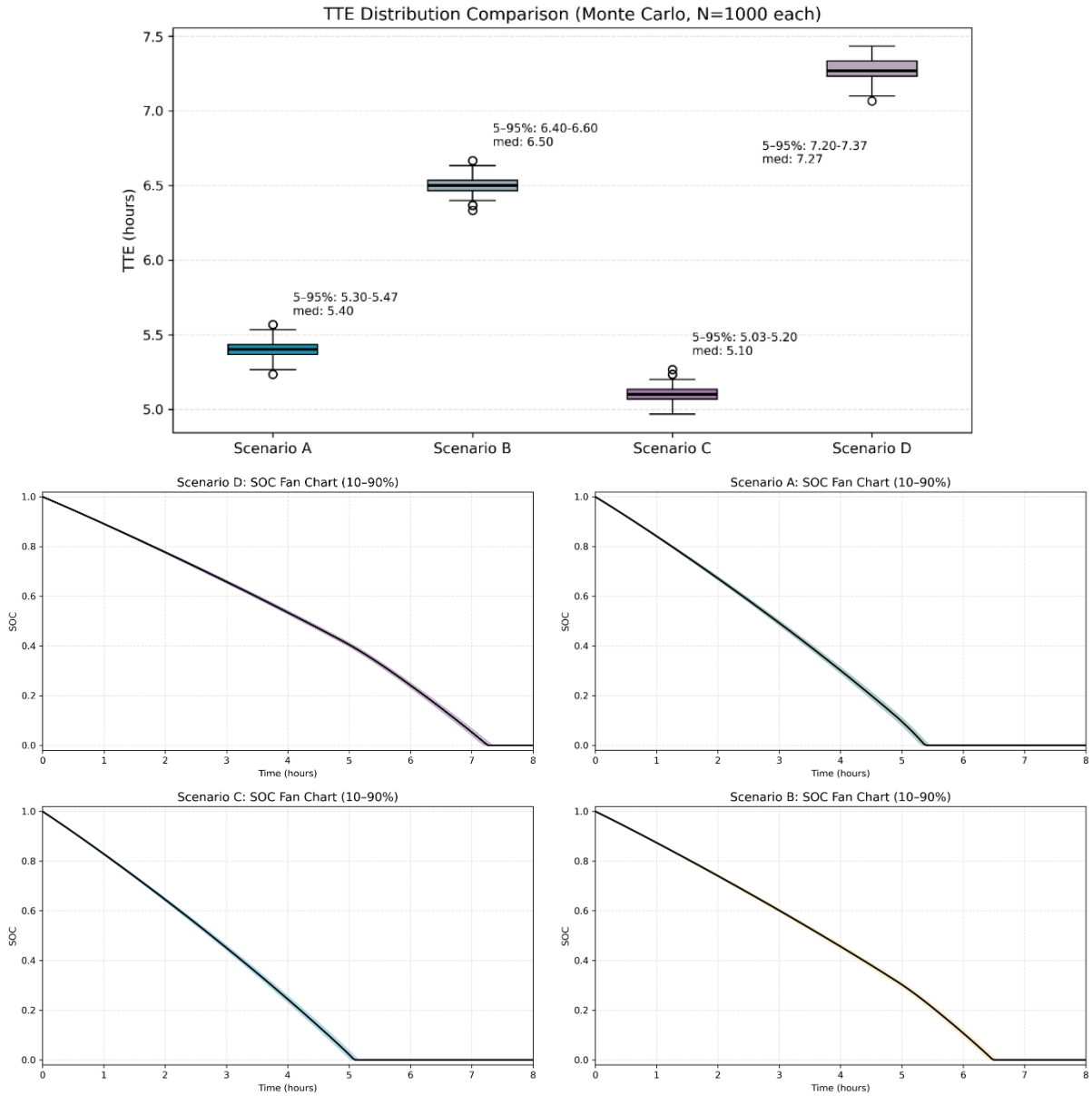
8.2 Robustness Check

To verify the stability and interpretability of the model under realistic uncertainties, we conduct a robustness analysis. This analysis focuses on whether the model's predictions for TTE and SOC curve remain consistent amidst usage pattern fluctuations and parameter uncertainties.

We employ a **Monte Carlo framework**: given specific hardware and battery states, we simulated multiple power consumption trajectories. Each trajectory is described by a **piecewise stationary random process**, where key parameters fluctuate around typical levels within different time segments. For each trajectory, the model iteratively updates the battery level at discrete time steps (Δt):

$$Q(t+\Delta t) = Q(t) - I_{battery} \cdot \frac{\Delta t}{3600}$$

By repeating the simulation for N trajectories, we obtain the distribution of TTE (including mean, standard deviation, and percentile intervals) and the **uncertainty bands** for SOC.



As observed, the result intervals across the distinct random trajectories are **remarkably narrow**, and the SOC trajectories are tightly clustered around the mean. This **tight dispersion** indicates that the model predictions are highly insensitive to the introduced fluctuations, thereby validating the model's robust performance against usage variability and parameter uncertainties.

9 Model Evaluation and Usage Suggestions

9.1 Strengths

1.We have reasonably simplified the internal structure of the mobile phone. Despite the complexity of the motherboard and SoC, we construct an external circuit current equation by measuring the current during specific user behaviors (which are also common behaviors) and correlating them with user actions. We then describe the external circuit current at any given moment using a combination of multiple independent sub-currents.

2.The voltage regulator model describes the connection between the external circuit and the

power supply. The relationship between lithium battery voltage and SOC is approximated as a linearly positive correlation. These enable that key physical quantities such as heat generation and battery current can be represented.

3.We employe curve fitting methods to express segmented functions as uniformly formatted functions with negligible differences between them. This approach avoids cumbersome case-by-case analysis in subsequent computations.

9.2 Weaknesses

The Time-to-Empty Predictions Model relies heavily on the K-Means clustering algorithm to classify user behavior patterns, but the clustering result is sensitive to the initial cluster centers. If the initial parameters are set unreasonably, the user portrait classification will be inaccurate, which will affect the prediction effect of the remaining battery life under different user usage scenarios.

9.3 Usage Suggestions

9.3.1 Usage Suggestions and Practical Implementation

The following strategic recommendations are derived from our **Continuous-Time SOC Model** and **User Behavior Clustering**, bridging mathematical theory with practical device management:

- **Luminance Optimization**: Our model confirms a strong positive correlation between screen brightness (cd) and backlight current (I_{sc}). Users should utilize Auto-Brightness and limit manual settings to 30%–50% in dark environments. This minimizes I_{sc} and extends Time-to-Empty (TTE) while maintaining visual comfort.
- **Background Activity Management:** Since background application currents (I_{bi}) contribute significantly to the total load current (I_o), users should terminate dormant high-drain apps (e.g., GPS, Video Streaming). Thermal Regulation: * High Temp (>35°C): Avoid prolonged exposure to heat, as our aging model shows it exponentially accelerates SEI film thickening, leading to irreversible degradation.
- **Low Temp:** Operating in extreme cold spikes internal resistance (r), reducing effective usage time to 70%–80% of standard capacity.

9.3.2 Strategic Recommendations for Mobile Manufacturers

Manufacturers should integrate our Continuous-Time SOC Model into the Battery Management System (BMS). Unlike traditional sensors, this low-cost software solution utilizes historical interaction data and real-time parameters ($I_{battery}$,cd) to provide precise, personalized TTE forecasting.

We propose an Integrated Battery Health Monitoring (BHM) Module to track the evolution of full-charge capacity (Q_0), internal resistance (r), and SEI thickness. Providing a transparent "Health Score" and alerting users when $Q_0 < 80 < Q_{fs}$ ensures operational safety.

10 Conclusion

The mechanism of smartphone battery power consumption is a very complex issue in electrical engineering and energy science. So far, it is inconclusive, and it may take longer to get to the explanations. There are still many potential rules and optimization spaces waiting to be explored by researchers.

We hope that our model can provide some help for further study of smartphone battery power consumption behavior, and we also hope that readers can give us some suggestions to improve our model.

11 References

[1] OpenAI. ChatGPT (Version 5.2) [CP/OL]. OpenAI, 2026(2026-01-30). https://www.openai.com/chatgpt

[2] Jing, K., Ding, Z., Liang, K., et al. A Novel Imidazolium-based Additive to Improve Low-temperature Performance of Si/C Anodes [J]. Power System Technology, 2025, 49(08): 1598-1605.

[3] Tan, Z., Sun, R., Yang, R., et al. Review on the Development of Battery Management Systems [J]. Journal of Chongqing Institute of Technology (Natural Science), 2019, 33(09): 40-45.

[4] Xu, P., Zhang, R., & Ding, F. Research on Calendar Aging of Lithium-ion Batteries Based on Electrochemical-Thermal Coupling Model [J]. Power System Technology, 2025, 49(08): 1638-1649.

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[7] Qi, Y., Wang, J., Chen, H., et al. Capacity Degradation Prediction of Lithium-ion Battery Calendar Aging Based on Double-layer Optimization [J]. Clean Coal Technology, 2025, 31(S2): 1062-1073.

[8] Tan, Z., Sun, R., Yang, R., et al. Review on the Development of Battery Management Systems [J]. Journal of Chongqing Institute of Technology (Natural Science), 2019, 33(09): 40-45.

[9] Detailed empirical testing protocols and raw data outputs are provided in Appendix 1. Throughout the remainder of this report, the phrase "consistent with experimental results" (or similar terminology) is utilized to indicate that model predictions have been cross-verified

against these empirical benchmarks without further individual citation.

[10] Wang, D. Research on Liquid Cooling and Optimization of 18650 Lithium Battery [D]. Huainan: Anhui University of Science and Technology, 2024.

[11] Theiler, M., Endisch, C., & Lewerenz, M. Float Current Analysis for Fast Calendar Aging Assessment of 18650 Li(NiCoAl)O2/Graphite Cells [J]. Batteries, 2021, 7(22). https://doi.org/10.3390/batteries7020022

Data resources

[1] https://www.kaggle.com/datasets/valakhorasani/mobile-device-usage-and-user-behavior-dataset

[2] https://www.kaggle.com/datasets/prince7489/smartphone-usage-dataset

[3] https://huggingface.co/datasets/Battery-Life/BatteryLife_Raw/tree/main/CALB/45%E5%BA%A6

[4] Wuhan Jisu Shikong Technology Co., Ltd. Xiaobai Evaluation Database [EB/OL]. Accessed via WeChat Mini Program: Xiaobai Database, 2025(2026-02-02). (ICP Filing No. Hubei ICP 2025118950-1X)

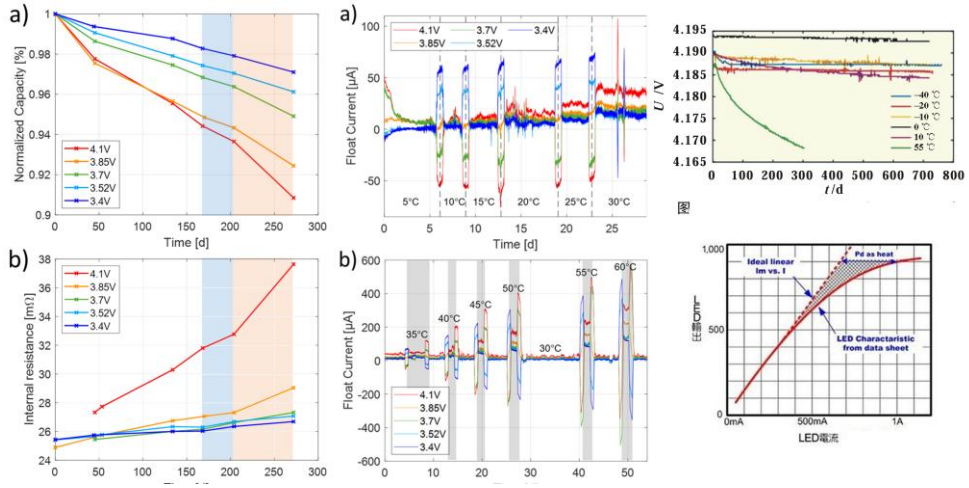
[5] Shanghai Xiancun Cultural Communication Co., Ltd. SOCPK - Geekerwan Mobile Chip Ranking [EB/OL]. Geekerwan, 2025(2026-02-02). https://www.socpk.com/. (ICP Filing No. Shanghai ICP 2024064919)

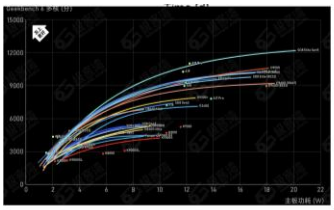
Appendices

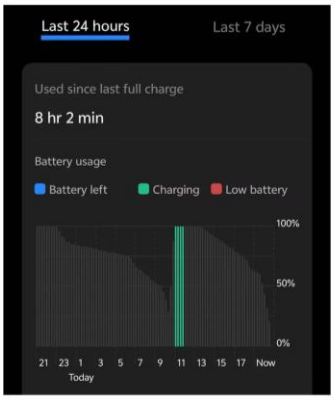
Appendix 1

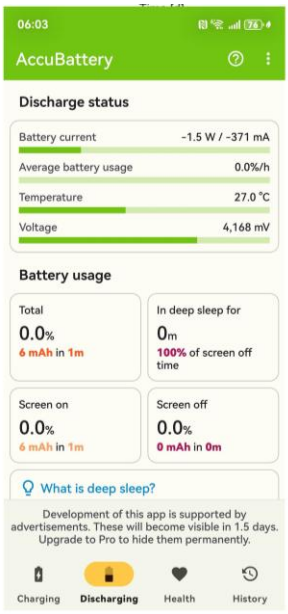
Data-sets and Examinations

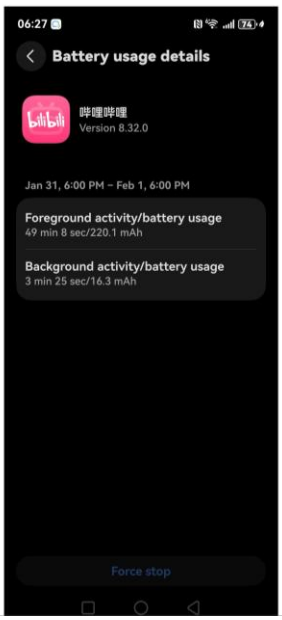
Some of our Examinations and datasets.





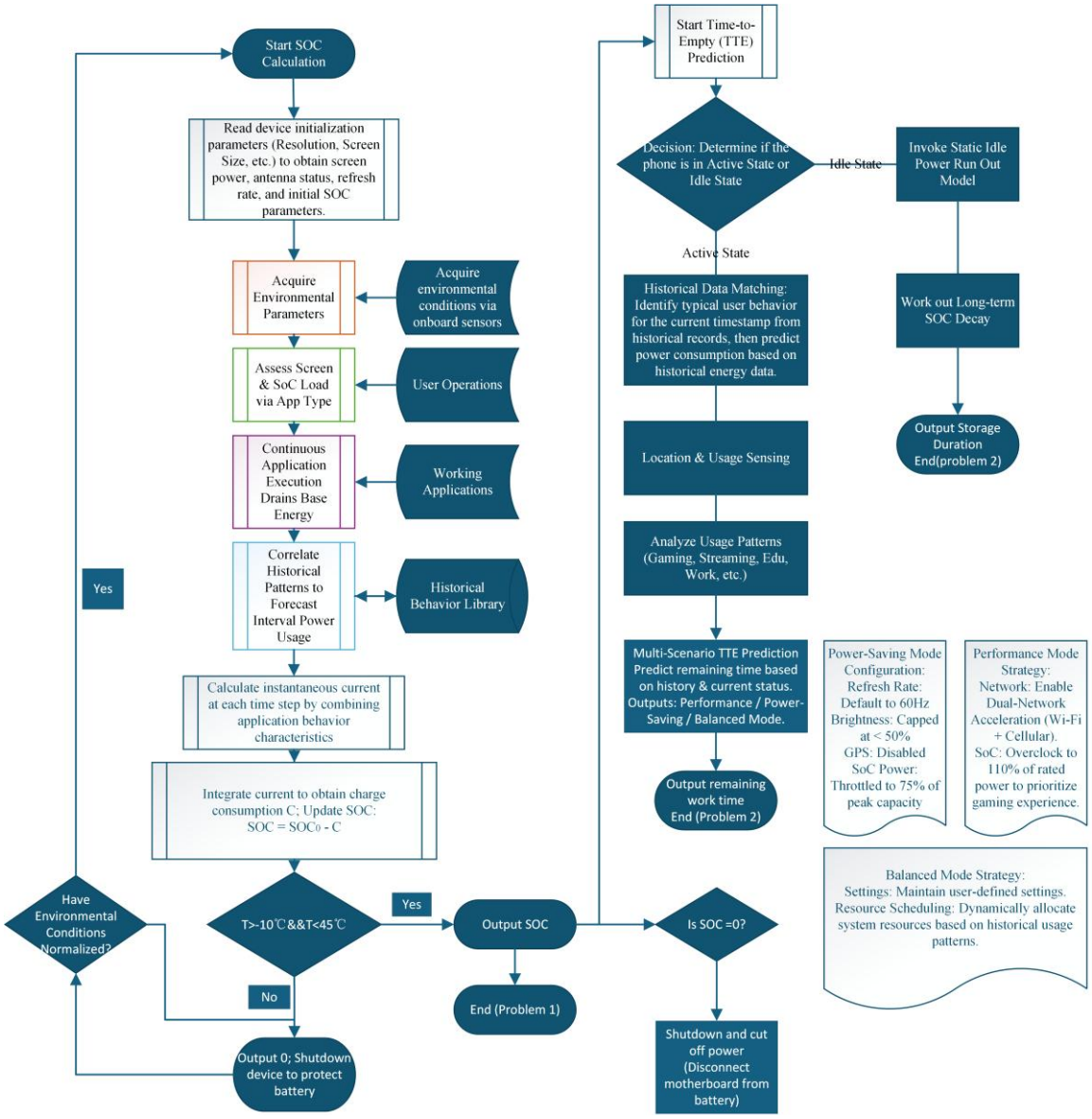






Appendix 2

Total model flow chart



Report on Use of AI Tools

1.Google Gemini, Google (Nov. 11, 2025 version, Gemini 3 Pro)

Date of Use: February 1 – February 2, 2026
Description of Use: We utilized Google Gemini 3 Pro as a sophisticated translation and linguistic refinement tool. Specifically, the AI was employed to:

1. **Technical Translation:** Convert the primary manuscript from its native language into English, ensuring the preservation of the original logical framework.
2. **Lexical Optimization**: Refine the academic tone and terminology to align with international standards. This included strictly differentiating between the acronyms "SOC" (State of Charge) and "SoC" (System on Chip) to avoid ambiguity, as well as utilizing professional terms like "Empirical Validation" and "Differential Decomposition."
3. **Formatting Alignment:** Assist in restructuring the text to adhere to the formal MCM/ICM Summary Sheet and paper formatting requirements.

2.ChatGPT, Open AI (Dec. 11, 2025 version, ChatGPT 5.2)

Tool Used: ChatGPT
Developer: Open AI
Date of Use: January 30 – February 1, 2026

Query 1:

I want to draw a simple flow chart with a nature-style, help me with the color matching, and provide 4-6 harmonious blue color schemes (displayed in hexadecimal RGB format).

Output 1:

Sure thing ～

Below are 4 sets of minimalist blue color schemes (5 colors per set) with a Nature style. They are overall natural, clean and soft, very suitable for flow charts / info graphics / product logic diagrams, and all of them are very harmonious without being harsh to the eyes.

Scheme 1 | Morning Lake Blue (Gentle & Healing)Suitable for: Concept flow charts, lightweight logic diagrams• Main Color (Nodes): #4A90A4• Secondary Main Color (Emphasized Nodes): #6FB7C8• Auxiliary Color (Connections): #A7DADC•

Light Background: #EAF6F6• Dark Text / Borders: #2F4F5F It feels like lake water and mist, very natural.

Scheme 2 | Forest Sky Blue (Natural and Stable) Suitable for: Professional/product flow charts • Main Color: #3A6F8F • Secondary Main Color: #5B8FB9 • Auxiliary Color: #8FBBD9 • Light Background: #EEF4F8 • Dark Emphasis: #2B4A5A It has a touch of forest atmosphere in the blue and is not too cold.

Scheme 3 | Minimalist Misty Blue (High-end and Space-friendly) Suitable for: Minimalist UI/high-end flow charts • Main Color: #5F8FA3 • Secondary Main Color: #7FAFC2 • Auxiliary Color: #BFDCE5 • Background Color: #F5FAFC • Text/Key Points: #3A5F6F It is very suitable for flow charts on white background with clean lines.

Scheme 4 | Coastal Natural Blue (Slightly Lively) Suitable for: User journeys/multi-step processes • Main Color: #2F7E9E • Secondary Main Color: #4FA3C2 • Auxiliary Color: #9ED0E0 • Light Background: #ECF7FA • Dark Emphasis: #1F4E63 It is more lively but still natural and not gaudy.

Useful Tips • Node Filling: Main Color/Secondary Main Color • Connections & Arrows: Auxiliary Color (70–80% opacity for a more natural look) • Background: Light Background Color • Text: Dark Emphasis (85% opacity is optional) If you want, I can also: • Directly specify the complete flow chart color matching rules for "nodes/connections/highlights/background" • Customize a set according to the usage scenarios of Fig Jam/Fig ma/PPT/Mermaid • Adjust to be "more natural/more technological/more Japanese-style"

Which tool do you plan to use for drawing?

query 2:

I am writing a mathematical modeling paper. I am not a native English speaker and have little knowledge of the format of academic papers. Please provide me with some advice on writing the paper, which can be given from aspects such as format, tone, and how to typeset text and match pictures.

Output 2:

Of course!

I will try my best to stand from the perspective of "someone who is writing an academic paper for the first time and is not a native English speaker" to provide you with a practical, step-by-step writing guide, focusing on mathematical modeling papers (applicable to both competition papers and course papers).

I. Standard Structure of a Mathematical Modeling Paper (Follow this, and you will hardly go wrong)

A typical mathematical modeling paper usually includes the following parts (the order is very important):

1 Title

• Concise + Accurate

• Try to explain "what you did" and "what method you used"

Example:

A Mathematical Model for Optimizing Urban Traffic Flow Based on Queuing Theory

Suggestions:

• Do not be too fancy

• Use fewer abbreviations

• Generally no more than 15–20 words

2 Abstract ★ Very Important

The abstract is usually 150–300 words with a fixed structure:

Four-sentence logic (Highly Recommended):

1. What is the research background / problem?

2. What model was established / what method was used?

3. What problem was solved / what results were obtained?

4. What is the significance / advantage of the results?

Common Sentence Templates:

This paper studies …

A mathematical model based on … is established to …

The results show that …

The proposed model is effective in …

Notes:

• No formulas

• No reference to charts or figures

• Use simple present tense / simple past tense

3 Keywords

• Generally 3–6 keywords

• Use nouns or noun phrases

Example:

Keywords: Mathematical modeling; Optimization; Differential equations; Simulation

4 Introduction

Function: Tell readers what you are doing & why it is worth doing

Structure Suggestions:

1. Problem background (practical significance)

2. Brief description of relevant research (not too detailed)

3. This work (what you did)

4. Article structure arrangement

Common Sentences:

In recent years, … has attracted increasing attention.

However, existing studies mainly focus on …

In this paper, we propose …

Do not start with formulas directly!

5 Assumptions and Symbols ★ Feature of Modeling Papers

Must be a separate section.

Assumptions

• Use numbered lists

• Assumptions should be reasonable, clear, and explainable

Example:

Assumption 1: The system operates under steady-state conditions.

Assumption 2: External disturbances are ignored.

Symbols

• A table is recommended

Symbol	Description
x	Decision variable
t	Time
λ	Arrival rate

6 Model Establishment ★ Core Section

Structure Suggestions:

• Problem decomposition

• Variable definition

• Model formulas

• Model explanation (Very Important!)

Key Principles:

Formulas must be explained with words!

Wrong Example (Common Pitfall):

The model is shown as follows:

$f(x) = ax + b$

☑ Correct Example:

Based on the above assumptions, the objective function is established as:

$f(x) = ax + b$

where a represents … and b denotes …

7 Model Solution / Algorithm

• Analytical solution? Numerical solution? Simulation?

• Clearly write the method + steps

If it is an algorithm:

• A flow chart can be used

• Or pseudo code (Algorithm 1)

The simple past tense can be appropriately used here.

8 Results and Analysis

This is the section with the most figures.

Suggested Structure:

• Result presentation (figures/tables)

• Result explanation

• Sensitivity analysis / Comparative analysis

Principles:

• One figure with one explanation

• Figures are not decorations; they must be explained.

9 Discussion (Optional)

• Advantages of the model

• Limitations

• Scope of application

Conclusion

• Summarize the model

• Emphasize the results

• Outlook on future work

Do not introduce new content!

II. English Tone & Writing Style (Most Important for Non-Native Speakers)

✓ Recommended Tone

• Formal

• Objective

• Third person

• Minimize the use of "I / we" (some journals allow "we")

✗ Avoid

• Colloquial expressions (a lot of, very, really)

• Emotional words (amazing, terrible)

• Long sentences nested within long sentences

III. Typesetting of Formulas, Figures, and Tables (Valued by Teachers and Judges)

Formulas

• On a separate line

• Numbered right-aligned (1), (2), …

• Citation in the text: Equation (3)

Figures

• Figure captions below the figures

• Figures should be clear; fonts should not be too small

• Must be cited in the main text

Example:

Figure 2 shows the variation of … with respect to time.

Tables

• Table captions above the tables

• Units clearly stated

• Vertical lines are preferably not used

IV. Practical "Survival Skills" Especially Useful for Mathematical Modeling Papers

☑ Use these words more

• establish

• propose

• derive

• analyze

• validate

• demonstrate

❌ Use these words less

• think

• believe

• feel

• guess

V. A Super Practical Suggestion for You 💟

c First, clarify the logic in Chinese, then translate it into English.

Or: Chinese → Simple English → Academic Polishing

If you want, I can help you with the following things next:

• Provide you with an English template for mathematical modeling papers (fill in the blanks directly)

• Help you revise the paragraphs you have already written into authentic academic English

• Specifically teach you "how to explain formulas without being verbose"

Which section are you writing now? I can directly help you revise from your current content ✨